Balancing riderless e-scooters at zero speed

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Abstract

Since e-scooters provide a great solution for the "first and last mile problem", they are very popular nowadays in road transportation. Unfortunately, they are often scattered in the streets, since many users leave them in the middle of the walkways. As a futuristic solution, we propose that e-scooters could drive themselves to docking stations or designated parking areas. In this study, the dynamics of riderless electric scooters is analyzed via a spatial mechanical model. The e-scooter is balanced at zero speed, by applying internal steering torque to the handlebar. A hierarchical, linear state feedback controller is designed with feedback delay. The linear stability charts of the delayed controller are constructed with semi-discretization. The effect of the center of gravity position of the handlebar on the linear stability is investigated.

Mechanical model and governing equations

The investigated spatial mechanical model of Fig. 1(a) is based on the Whipple bicycle model [1]. The multibody system consists of four rigid bodies: the handlebar and fork assembly, the body (the frame), and the front and rear wheels.

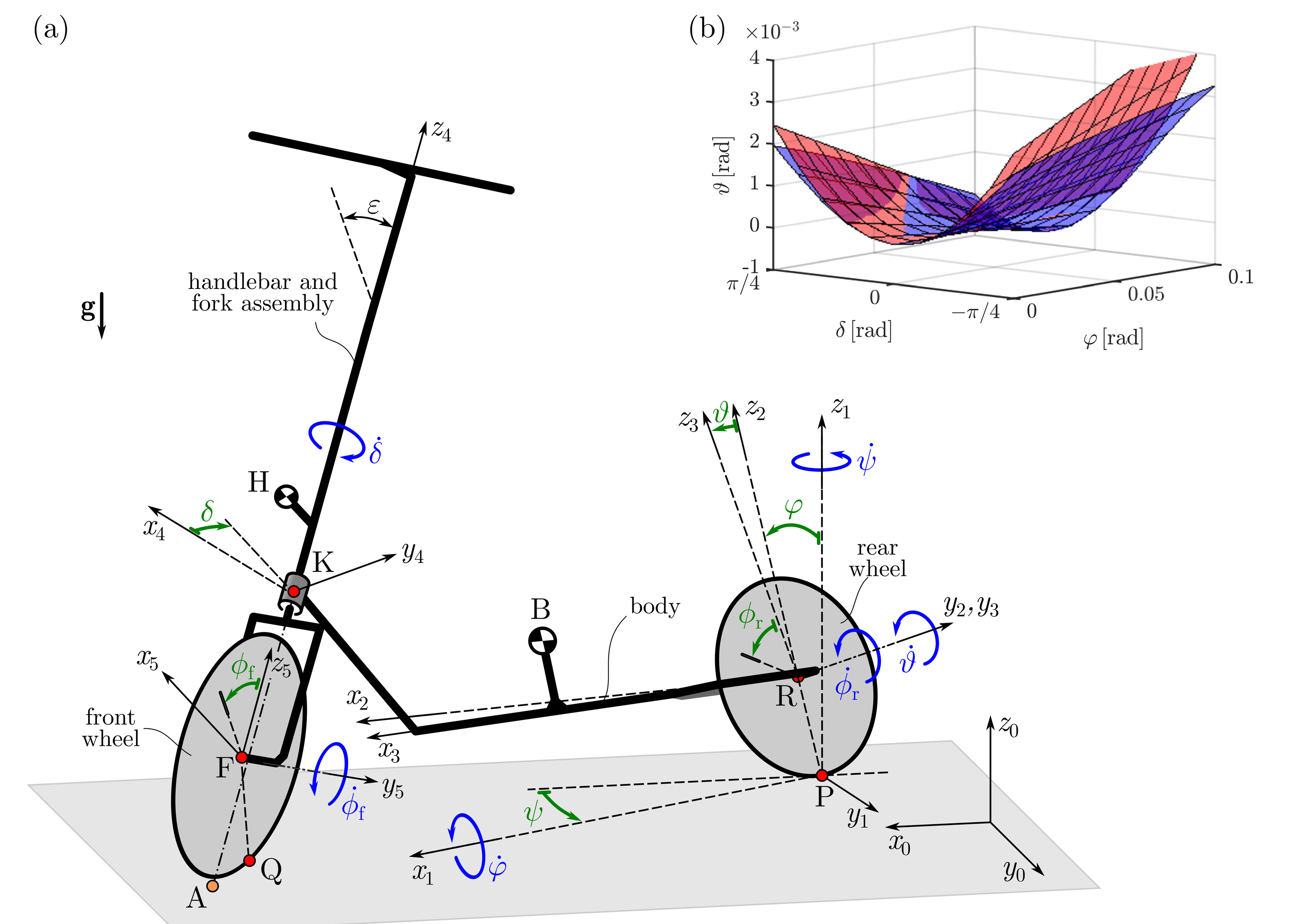

Figure 1: (a) The spatial model of an e-scooter. (b) The surface corresponding to the constraint equation for the pitch angle ϑ together with the approximate surface of Eq. (1).

Geometric constraints and generalized coordinates

- Three hinges between the bodies constrain three translational and two rotational DoF each, leading to 15 scalar constraining equations.
- The front and the rear wheels are attached to the ground leading to two additional geometric constraints in the system.
- The configuration space is $4 \cdot 6 - 17 = 7$ dimensional, i.e., one has to choose seven generalized coordinates. Let us choose the coordinates X and Y of the center point R of the rear wheel, the yaw angle ψ, the lean angle φ, the steering angle δ, and the rotational angles ϕ_f and ϕ_r of the front and the rear wheels around their rotational axes. The vector of generalized coordinates is $\mathbf{q} = \begin{bmatrix} X \; Y \; \psi \; \varphi \; \delta \; \phi_\mathrm{f} \; \phi_\mathrm{r} \end{bmatrix}^\mathrm{T}$.
- The pitch angle ϑ can be suppressed as the function of the lean and the steering angles. This provides a quartic equation for $\sin\vartheta$, which can be solved analytically but is cumbersome. To simplify the derivation of the linearized equation of motion, one can use an approximation as

$$\vartheta \approx \frac{e}{4p}\delta^2 \sin 2\varepsilon - \frac{e}{p}\varphi\delta\cos\varepsilon \,, \qquad (1)$$

where e is the trail, p is the wheelbase (the distance between the front and the rear wheel contact points) and ε is the rake angle, see Fig. 2(b).

Kinematic constraints and pseudo velocities

- We assume that the wheels roll purely on the flat ground. Altogether, four scalar kinematic constraining equations can be formulated for the two wheels, one longitudinal and one lateral for each wheel-to-ground contact.
- Since our goal is to stabilize the motorcycle for zero longitudinal speed, the rotational speed of the front wheel is also considered to be zero: $\dot\phi_\mathrm{f} = 0$.
- The number of pseudo velocities is equal to the difference between the number of the generalized coordinates and the number of kinematic constraints, i.e., $7 - 5 = 2$ in this study. The vector of pseudo velocities is $\boldsymbol{\sigma} = \begin{bmatrix} \dot\varphi \; \dot\delta \end{bmatrix}^\mathrm{T}$.

Equations of motion

- The nonlinear equations of motion can be derived considering the geometric and kinematic constraints of the system, e.g., with the help of Kane's method [2].
- The linearized equations of motion can be written as $\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{Q}$, where $\mathbf{M}$ is the mass matrix, and $\mathbf{K}$ is the stiffness matrix. The vector of state variables is $\mathbf{x} = \begin{bmatrix} \varphi \; \delta \end{bmatrix}^\mathrm{T}$. The vector of generalized forces is $\mathbf{Q} = \begin{bmatrix} 0 \; M^\mathrm{s} \end{bmatrix}^\mathrm{T}$ with internal steering torque M^s.
- The above-described linearized equations of motion agree with the literature [3] for zero speed.

Hierarchical linear state feedback controller

- We try to balance the e-scooter by using the steering mechanism, i.e., applying steering torque M^s on the handlebar.
- A higher-level controller calculates the desired steering angle as

$$\delta_\mathrm{des} = -K^\mathrm{s}_{\mathrm{p}\varphi}\varphi(t-\tau) - K^\mathrm{s}_{\mathrm{d}\varphi}\dot\varphi(t-\tau)\,, \qquad (2)$$

where τ is the feedback delay of the controller.
- The internal steering torque is created by a lower-level control law as

$$M^\mathrm{s} = -K^\mathrm{s}_{\mathrm{p}\delta}\left(\delta(t) - \delta_\mathrm{des}\right) - K^\mathrm{s}_{\mathrm{d}\delta}\dot\delta(t)\,. \qquad (3)$$

- For different lower-level control gain pairs $(K^\mathrm{s}_{\mathrm{p}\delta}, K^\mathrm{s}_{\mathrm{d}\delta})$, stability charts were constructed by semi-discretization [4] in the plane of the higher-level control gains and the optimum points were obtained. Then, a so-called stabilizability plot was constructed, namely, the real part of the rightmost characteristic exponent $\mathrm{Re}\lambda_\mathrm{max}$ related to the optimum point is plotted in the plane of the lower-level control gains $K^\mathrm{s}_{\mathrm{p}\delta}$ and $K^\mathrm{s}_{\mathrm{d}\delta}$, see Fig. 2(a). The optimal lower-level control gain pair is $K^\mathrm{s}_{\mathrm{p}\delta} = 10\,\mathrm{Nm}$ and $K^\mathrm{s}_{\mathrm{d}\delta} = -5\,\mathrm{Nms}$, see the black cross.

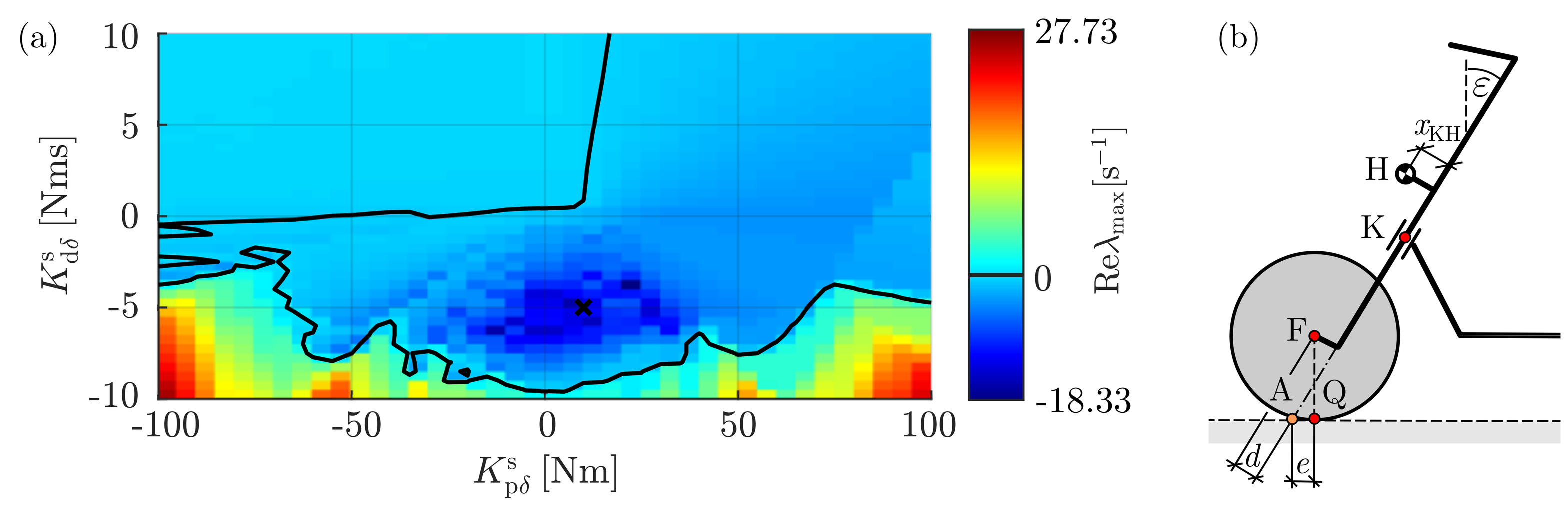

Figure 2: (a) Stabilizability plot: the real part of the rightmost characteristic root $\mathrm{Re}\lambda_\mathrm{max}$ for lower-level control gain pairs $(K^\mathrm{s}_{\mathrm{p}\delta}, K^\mathrm{s}_{\mathrm{d}\delta})$. (b) The geometric parameters that have a relevant effect on the linear stability.

Parameters with relevant effects on the linear stability

According to previous research, the feedback delay τ, the rake angle ε, the trail e and the center of gravity of the handlebar x_KH have significant effects on the linear stability properties, see Fig. 2(b). The more the center of gravity of the handlebar afore the steering axis is, the greater the linearly stable region is, see the linear stability charts of Fig. 3(a). However, the vertical position can only be stabilized for $x_\mathrm{KH} < 0.0277\,\mathrm{m}$, see Fig. 3(b). The optimum value for the center of gravity of the handlebar is approximately $x_\mathrm{KH}{}^\mathrm{opt} = 0.0106\,\mathrm{m}$.

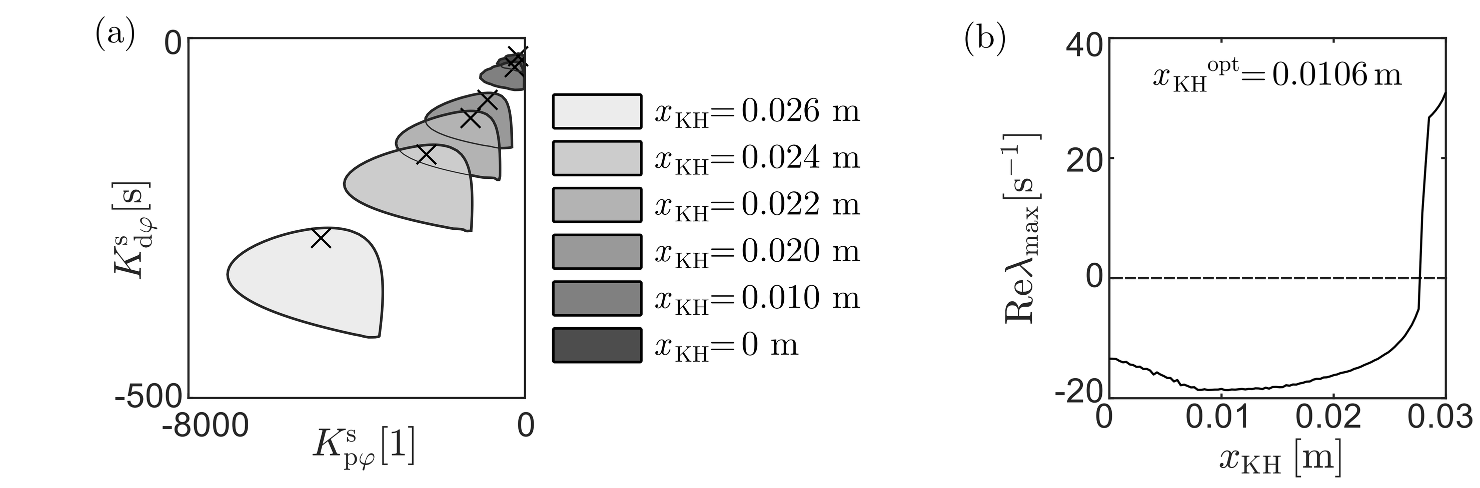

Figure 3: The effect of the center of gravity of the handlebar x_KH on the linear stability for fixed lower-level control gains $K^\mathrm{s}_{\mathrm{p}\delta} = 10\,\mathrm{Nm}$ and $K^\mathrm{s}_{\mathrm{d}\delta} = -5\,\mathrm{Nms}$ and fixed geometric parameters [5]. (a) Linear stability charts obtained by semi-discretization [4], (b) the real part of the rightmost characteristic roots by means of the center of gravity of the handlebar x_KH.

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